

# Relativistic Study of Mesonic Baryon Resonance Decays

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**Abstract.** Mesonic baryon resonance decays are calculated from constituent quark models along a Poincaré-invariant generalization of the elementary emission model. Covariant results of pionic decay widths are presented for the Goldstone-boson-exchange constituent quark model.

## 1 Introduction

Investigations of mesonic resonance decays have a long tradition, and in the focus of interest have been, notably, the performances of various constituent quark models (CQMs) as well as the adequacy of different decay operators for the mechanism of meson creation/emission. Despite considerable efforts invested one has still not yet arrived at a satisfactory explanation especially of the  $N^*$  and  $\Delta$  resonance decays. Also complementary attempts beyond the CQM approach have not succeeded much better with hadron decays, and more generally, with providing a comprehensive working model of low-energy hadronic physics based on QCD. This situation is rather disappointing from the theoretical side, especially in view of the large amount of experimental data accumulated over the past years and the ongoing high-quality measurements at such facilities as JLAB, MAMI and others (for an overview of the modern developments see the proceedings of the past  $N^*$  Workshop [1]).

In a recent CQM the interaction between two constituent quarks was based on Goldstone-boson exchange (GBE) [2] in addition to a linear confinement. The so-called GBE CQM at the same time fulfills the requirements of Poincaré invariance and thus it paves the way to a relativistic treatment of reactions involving baryons. Specifically, due to the usage of a relativistic kinetic-energy operator, the Hamiltonian of the GBE CQM leads to an invariant mass operator in relativistic quantum mechanics, where the quark-quark interactions are introduced via the Bakamjian-Thomas construction [3].

Up till now the GBE CQM has already been put to some tests in calculating

mesonic decays of resonances of light and strange baryons in a non-relativistic framework [4, 5, 6]. Here, we calculate the mesonic decay widths in a covariant approach along a generalization of the elementary emission model (EEM) in point-form relativistic quantum mechanics [7]. We compare the predictions of the GBE CQM to analogous results from a CQM with a one-gluon-exchange (OGE) hyperfine interaction, namely the relativized Bhaduri-Cohler-Nogami CQM as parametrized in ref. [6].

## 2 Theory

Generally, the decay width of a particle is defined by the expression

$$\Gamma = 2\pi\rho_f |F(i \rightarrow f)|^2, \quad (1)$$

where  $F(i \rightarrow f)$  is the transition amplitude and  $\rho_f$  is the phase-space factor. In Eq. (1) one has to average over the initial and to sum over the final spin-isospin projections. In non-relativistic calculations of baryon resonance decays one has usually made an arbitrary choice of the phase-space factor. In the rest frame of the decaying resonance, either a non-relativistic form,  $\rho_f = 2\pi \frac{M_f M_\pi}{M_i} q$ , or a

**Table 1.** Predictions for pionic decay widths by the GBE CQM [2] along the EEM in PFSA in comparison to experiment, an analogous calculation with the OGE CQM [6], and results from a non-relativistic EEM approach.

Decays	Experiment [8]	Rel. PFSA		Nonrel. EEM	
		CQM		GBE CQM	
		GBE	OGE	dir	dir+rec
$N_{1440}^* \rightarrow \pi N_{939}$	$(227 \pm 18)_{-59}^{+70}$	30.3	37.1	4.85	6.16
$N_{1520}^* \rightarrow \pi N_{939}$	$(66 \pm 6)_{-5}^{+9}$	16.9	16.2	22.0	38.3
$N_{1535}^* \rightarrow \pi N_{939}$	$(67 \pm 15)_{-17}^{+55}$	93.2	122.8	24.3	574.3
$N_{1650}^* \rightarrow \pi N_{939}$	$(109 \pm 26)_{-3}^{+36}$	28.8	38.3	11.3	160.3
$N_{1675}^* \rightarrow \pi N_{939}$	$(68 \pm 8)_{-4}^{+14}$	5.98	6.20	7.65	15.1
$N_{1700}^* \rightarrow \pi N_{939}$	$(10 \pm 5)_{-3}^{+3}$	0.91	1.19	1.43	2.87
$N_{1710}^* \rightarrow \pi N_{939}$	$(15 \pm 5)_{-5}^{+30}$	4.06	2.28	23.4	5.95
$\Delta_{1232} \rightarrow \pi N_{939}$	$(119 \pm 1)_{-5}^{+5}$	33.7	32.1	59.1	81.2
$\Delta_{1600} \rightarrow \pi N_{939}$	$(61 \pm 26)_{-10}^{+26}$	0.116	0.503	74.2	55.7
$\Delta_{1620} \rightarrow \pi N_{939}$	$(38 \pm 8)_{-6}^{+8}$	10.4	14.6	4.82	74.8
$\Delta_{1700} \rightarrow \pi N_{939}$	$(45 \pm 15)_{-10}^{+20}$	2.92	3.10	7.12	14.4

relativistic form  $\rho_f = 2\pi \frac{E_f E_\pi}{M_i} q$  has been used. Herein,  $M_i$  is the mass of the initial state and  $M_f, M_\pi$  as well as  $E_f, E_\pi$  are the masses and energies of the decay products, the nucleon and the pion, respectively. In the present work we follow a Poincaré-invariant description of the transition amplitude resulting in a unique choice of the phase-space factor.

For the actual calculation in the point form it is advantageous to introduce so-called velocity states

$$|v; \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3; \mu_1, \mu_2, \mu_3\rangle = U_{B(v)} |k_1, k_2, k_3; \mu_1, \mu_2, \mu_3\rangle = \prod_{i=1}^3 D_{\sigma_i \mu_i}^{\frac{1}{2}} [R_W(k_i, B(v))] |p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3\rangle, \quad (2)$$

where  $B(v)$  is a boost with four-velocity  $v$  and  $U_{B(v)}$  its unitary representation. The boosted momenta are defined by  $p_i = B(v) k_i$ , where  $k_i = (\omega_i, \mathbf{k}_i)$  and  $\sum \mathbf{k}_i = 0$ ; the  $D^{\frac{1}{2}}$  are the spin- $\frac{1}{2}$  representation matrices of Wigner rotations  $R_W(k_i, B(v))$ . The baryons are described by simultaneous eigenstates of the four-momentum operator (or equivalently the mass operator), the total-angular-momentum operator, and its z-component; we denote them by  $|P, J, \Sigma\rangle$ . The transition amplitude is then defined in a Poincaré-invariant fashion, under overall momentum conservation ( $P' - P = Q_\pi$ ), by

$$\begin{aligned} F(i \rightarrow f) &= \langle P, J, \Sigma | \hat{\mathcal{D}}_\alpha | P', J', \Sigma' \rangle \\ &\sim \int d^3 k_2 d^3 k_3 d^3 k'_2 d^3 k'_3 \\ &\quad \Psi_{MJ\Sigma}^* (\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3; \mu_1, \mu_2, \mu_3) \Psi_{M'J'\Sigma'} (\mathbf{k}'_1, \mathbf{k}'_2, \mathbf{k}'_3; \mu'_1, \mu'_2, \mu'_3) \\ &\quad \prod_{\sigma_i} D_{\sigma_i \mu_i}^{\frac{1}{2}\star} [R_W(k_i, B(v_{in}))] \prod_{\sigma'_i} D_{\sigma'_i \mu'_i}^{\frac{1}{2}} [R_W(k'_i, B(v_f))] \\ &\quad \langle p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 | \hat{\mathcal{D}}_\alpha | p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 \rangle, \end{aligned} \quad (3)$$

where the baryon wave functions  $\Psi_{MJ\Sigma}^*$  and  $\Psi_{M'J'\Sigma'}$  enter as velocity-state representations of the baryon states  $\langle P, J, \Sigma |$  and  $|P', J', \Sigma'\rangle$ , respectively. In a first attempt, we investigate a decay operator, which can be interpreted as a generalization of the EEM. Namely, we assume that a pion is created on one of the quarks, while the other two quarks of the decaying baryon resonance are merely spectators. Consequently, the decay operator is taken in point-form spectator approximation (PFSA). Here, we assume a pseudo-vector coupling for pointlike particles. This yields the decay operator in the form

$$\begin{aligned} &\langle p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 | \hat{\mathcal{D}}_\alpha | p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 \rangle \\ &\sim 3ig_{q\pi} \bar{u}(p_1, \sigma_1) \gamma^5 \gamma^\mu \boldsymbol{\lambda}^F u(p'_1, \sigma'_1) \\ &\quad \times 2p_2^0 \delta(\mathbf{p}_2 - \mathbf{p}'_2) 2p_3^0 \delta(\mathbf{p}_3 - \mathbf{p}'_3) \delta_{\sigma_2 \sigma'_2} \delta_{\sigma_3 \sigma'_3} Q_\mu, \end{aligned} \quad (4)$$

where  $g_{q\pi}$  is the pion-quark coupling constant and  $\boldsymbol{\lambda}^F$  the flavour operator. It should be noted that in PFSA the impulse delivered to the quark that emits

the pion is not equal to the impulse delivered to the baryon as a whole. The momentum transfer  $\tilde{q}$  to this single quark is uniquely determined from the momentum  $Q$  transferred to the baryon and the two spectator conditions.

### 3 Results

In Table 1 we present the predictions of the GBE CQM for pionic decay widths of  $N^*$  and  $\Delta$  resonances. They have been calculated directly from the CQM wave functions with the EEM decay operator in PFSA; no further parametrization has been introduced. As is immediately evident, only in a few instances a reasonable agreement with experiment is found. In most cases the decay widths remain far from the data, with the theoretical values always being smaller than the measured ones. These characteristics of the relativistic decay widths are in contrast to the ones calculated along the EEM in a non-relativistic approach plus first-order relativistic (recoil) corrections [4, 5]. On the other hand, the OGE and GBE CQMs produce quite similar results indicating that dynamical effects from the CQM are of lesser importance than relativistic effects.

In any case, we find the relativistic results to exhibit a completely new behaviour. This might largely be due to Lorentz boost effects, which have here been included exactly. Certainly, the predictions for the pionic decay widths are by no means satisfactory. This hints to persisting deficiencies in the decay operator and/or the CQM wave functions. The latter, relying on  $\{QQQ\}$  configurations only, are tentatively not realistic enough especially for the decaying resonance state. Further work will have to clarify these shortcomings.

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